9/19 Recitation

1. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the general solutions to $A\mathbf{x} = \mathbf{0}$ and $A\mathbf{x} = \mathbf{b}$. How do they compare?

2. Suppose A is an $m \times n$ matrix. Determine the number of potential solutions that $A\mathbf{x} = \mathbf{b}$ can have if:

   (a) $m \geq n$

   (b) $m < n$

   Explain your reasoning.

3. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.

   (a) Show that $T(\mathbf{0}) = \mathbf{0}$.

   (b) Show that the kernel of T is a subspace of $\mathbb{R}^n$ using the definition of subspace.

   (c) T is called **onto** if $\text{im}T = \mathbb{R}^m$ and **one-to-one** if $\ker T = \{0\}$. Show that if T is onto and one-to-one, then T is invertible (Hint: show that the matrix associated to T is invertible).

4. (a) Open the **Julia7SolutionSets** notebook to learn about a command that provides a spanning set of $N(A)$ and to see a simple function that provides the associated general solution.

   (b) **Coding Challenge**: Do coding challenge A in the notebook. At the bottom of the notebook, you can find a solution. As I am not a coder, my code may not be the most efficient or elegant. Feel free to improve on it!